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SHORTER COMMUNICATIONS

BOUNDARY-LAYER FLOW ON A FLAT PLATE

J. W. Rose
Department of Mechanical Engineering, Queen Mary College, University of London, England

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FOR STEADY, uniform-property, laminar, boundary-layer flow on an isothermal flat plate, with negligible dissipation and uniform free-stream conditions, the surface heat-transfer is given by:

$$Nu_x Re_x^{-1/2} = \zeta(Pr) \quad (1)$$

where Nu_x and Re_x are the local Nusselt and Reynolds numbers and Pr is the Prandtl number. Evans [1] has tabulated numerical solutions giving values of ζ for Prandtl numbers ranging from 0.0001 to 20000. The present note provides a simple formula which is a close fit to these data over the whole range of Prandtl number.

For $Pr >$ about 0.6, ζ is often, following Pohlhausen [2], approximated by:

$$\zeta(Pr) = 0.332 Pr^{1/3}. \quad (2)$$

When the data of Evans [1] are compared with equation (2) it is seen that the equation is correct for $Pr = 1$, is in error by about 1% for $Pr = 10$ and by about 2% for $Pr >$ about 20. For $Pr < 0.6$ the error soon becomes large as Pr decreases.

It may be seen from [1] that,

$$\text{for very large } Pr, \quad \zeta(Pr) \approx 0.339 Pr^{1/3}. \quad (3)$$

$$\text{for very small } Pr, \quad \zeta(Pr) \approx (1/\pi^{1/2}) Pr^{1/2}. \quad (4)$$

Thus, the expression:

$$\zeta(Pr) = \frac{Pr^{1/2}}{(A + BPr^C + DPr)^{1/6}} \quad (5)$$

would have the correct behavior for large and small Pr provided $0 \leqslant C \leqslant 1$.

Equation (5) was fitted to the values given by Evans [1] by minimizing the sum of squares of relative residuals of ζ. To avoid retaining excessive digits the constants were found in stages, at each of which one constant was rounded and fixed before redetermining the remaining constants. Equation (5), with the following (treated as exact):

$A = 27.8$
$B = 75.9$
$C = 0.306$
$D = 657$

has a maximum error of 0.33% over the range $0.0001 < Pr < 20000$.

The limiting values given by equation (5) and the above constants:

$$\text{for } Pr \to \infty, \quad \zeta(Pr) \to 0.339 Pr^{1/3} \quad (6)$$

$$\text{for } Pr \to 0, \quad \zeta(Pr) \to 0.575 Pr^{1/2} \quad (7)$$

may be compared with equations (3) and (4).

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REFERENCES

1. H. L. Evans, Laminar boundary-layer theory. Addison-Wesley, Reading, Massachusetts (1968).
2. E. Pohlhausen, Der Wärmeaustausch zwischen festen Körpern und Flüssigkeiten mit kleiner Reibung und kleiner Wärmeleitung, *Z. Angew. Math. Mech.* **1**, 115–121 (1921).

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HEAT-TRANSFER COEFFICIENT CORRELATIONS FOR THERMAL REGENERATOR CALCULATIONS—TRANSIENT RESPONSE

A. Burns
Computer Science and Systems Division, U.K. Atomic Energy Research Establishment, Harwell, Didcot, Oxfordshire OX11 0RA, U.K.

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NOMENCLATURE

A, regenerator heating surface area [m^2];
C, specific heat of storing matrix [J/kg K];
d, semi-thickness of wall of heat storing matrix [m];
E_{g1}, E_{g2}, dimensionless measure of transient response—continuous fit;
h, surface heat-transfer coefficient [W/m^2 K];
$\bar{h}$, bulk heat-transfer coefficient [W/m^2 K];
K, $N\phi/3$;
L, length of regenerator [m];
L_1, percentage measure of response accuracy;
M, mass of heat storing matrix [kg];
m, mass of gas resident in regenerator [kg];

N, Biot modulus, hd/λ;
P, period length [s];
S, specific heat of gas [J/kg K];
T, temperature of heat storing matrix [K];
t, temperature of gas [K];
ta, average inlet temperature of gas [K];
V, velocity of gas [m/s];
W, flow rate of gas [kg/s];
y, distance from regenerator entrance [m].

Greek symbols
α, exponent of flow rate, $h \propto W^{\alpha}$;
$\varepsilon_{g1}, \varepsilon_{g2}$, dimensionless measure of transient response;
η, dimensionless time;
Θ, total dimensionless time required to regain equilibrium following a step change in gas flow rate;
θ, time [s];
Λ, reduced length, $\bar{h}A/WS$ [dimensionless];
λ, thermal conductivity of heat storing matrix [W/m K];
ξ, dimensionless length;
Π, reduced period, $\bar{h}A(P-m/W)/MC$ [dimensionless];
ϕ, Hausen correction factor applied to account for the inversion of the parabolic solid temperature profile at the regenerator reversals;
δ, hydraulic diameter [m].

Subscripts
OUT, mean outlet;
0, prior to step change;
1, after step change.

Superscripts
$'$, refers to hot period;
$''$, refers to cold period;
(n), refers to cycle number n;
(0), refers to cyclic equilibrium prior to step change;
(∞), refers to cyclic equilibrium after step change.

THE DEPENDENCE OF BULK HEAT-TRANSFER COEFFICIENT UPON GAS FLOW RATE

Introduction

In two previous papers [1, 2] the transient behaviour of a thermal regenerator following a step change in operating conditions was investigated. Embodied in this work was the assumption that the overall or bulk heat transfer coefficient $\bar{h}$ is proportional to the mass flow rate W of the gas passing through the regenerator. This assumption enables changes in gas flow rate to be investigated by considering step changes in reduced period Π alone. The dimensionless parameters reduced length and reduced period can thus be used to study the effects of step changes in inlet gas temperature, gas flow rate and period duration.

The purpose of this paper is to examine the applicability of the assumption ($\bar{h} \propto W$) and to indicate the range of parameters for which its use is acceptable. The assumption of linear dependence between bulk heat transfer coefficient and gas flow rate in effect incorporates two distinct relationships, namely:

(i) The surface heat-transfer coefficient h is linearly proportional to gas flow rate, $h \propto W$.
(ii) The bulk heat-transfer coefficient (BHTC) and the surface heat transfer coefficient (SHTC) are similarly linearly proportional, $\bar{h} \propto h$.

In this paper relationships (i) and (ii) are considered generally, particular examples being used to illustrate the results expected from typical regenerator configurations, for example Cowper stoves.

Surface heat-transfer coefficients (SHTC)

There has been a considerable amount of experimental work on SHTC [3–6]; unfortunately the wide variety of materials and structures investigated has not enabled a "universal" relationship between SHTC and gas flow rate to emerge. Schofield *et al.* [3] recommend the use of the Kistner formulae for chequerworks of straight-through basket weave, equation (1) and double staggered, equation (2), design.

$$h = \frac{16.47V^{0.5}}{\delta^{0.333}}, \tag{1}$$

$$h = \frac{18.45V^{0.5}}{\delta^{0.333}}. \tag{2}$$

With the more common chimney type checkers Schofield *et al.* found the Bohm equations more accurate:

$$\text{For laminar flow, } h = ta^{0.25}\left(1.124 + \frac{0.375V}{\delta^{0.4}}\right). \tag{3}$$

$$\text{For turbulent flow, } h = \frac{0.903ta^{0.25}V^{0.8}}{\delta^{0.333}}. \tag{4}$$

In general the relationship between gas flow rate (usually operated turbulently) and SHTC can be written as:

$$h = FW^{\alpha} \tag{5}$$

where F and α are constants.

Mathematical model

The temperature behaviour of a thermal regenerator is represented by the differential equations (6) and (7) specified by Hausen [7].

$$\frac{\partial t}{\partial \xi} = T - t, \tag{6}$$

$$\frac{\partial T}{\partial \eta} = t - T, \tag{7}$$

where ξ and η are dimensionless measures of distance and time respectively:

$$\xi = \frac{\bar{h}Ay}{WSL}, \tag{8}$$

$$\eta = \frac{\bar{h}A}{MC}\left(\theta - \frac{my}{WL}\right). \tag{9}$$

Hausen proposed the dimensionless parameters "reduced length" Λ and "reduced period" Π to characterise each period of operation, where

$$\Lambda = \frac{\bar{h}A}{WS} \tag{10}$$

$$\Pi = \frac{\bar{h}A}{MC}\left(P - \frac{m}{W}\right). \tag{11}$$

The assumptions embodied in this model are discussed by Willmott and Thomas [8] and include the following physical idealisation:

The thermal conductivity of the matrix is zero in a direction parallel to that of the gas stream. In a direction perpendicular to gas flow, the thermal conductivity is considered either to be infinite, in which case the solid temperature is isothermal in this perpendicular direction, or to be finite. In the latter case a bulk heat transfer coefficient is developed which incorporates the surface resistance to heat transfer between gas and solid and the resistance internal to the heat storing solid.

With finite perpendicular conductivity Hausen [9] gives the following expression for the bulk heat-transfer coefficient $\bar{h}$:

$$\frac{1}{\bar{h}} = \frac{1}{h} + \frac{d\phi}{3\lambda} \tag{12}$$

where ϕ is used to take account of the effect of the solid parabolic temperature profile within the solid packing which inverts at the start of each period.

Estimation of the error produced by assuming $\bar{h} \propto W$

Let a regenerator initially at cyclic equilibrium be subject to a step change in gas flow rate from W_0 to W_1. The corresponding new SHTC is obtained from equation (5) namely

$$h_1 = h_0\left(\frac{W_1}{W_0}\right)^{\alpha}. \tag{13}$$

Similarly equation (12) yields the new BHTC, $\bar{h}_1$

$$\bar{h}_1 = \bar{h}_0\frac{h_1}{h_0}\left(1+\frac{h_0\,\mathrm{d}\phi}{3\lambda}\right)\left(1+\frac{h_1\,\mathrm{d}\phi}{3\lambda}\right)^{-1}. \tag{14}$$

Eliminating h_1 between equations (13) and (14) gives

$$\bar{h}_1 = \bar{h}_0\left(\frac{W_1}{W_0}\right)^{\alpha}(1+K)[1+K(W_1/W_0)^{\alpha}]^{-1}, \tag{15}$$

where $K = h_0\,\mathrm{d}\phi/3\lambda = N\phi/3$.

The assumption that $\bar{h} \propto W$ is equivalent to having $K = 0$ and $\alpha = 1$ in equation (15), that is:

$$\bar{h}_1^* = \bar{h}_0\frac{W_1}{W_0}, \tag{16}$$

where $\bar{h}_1^*$ is the value of the new BHTC obtained by the simplifying assumption.

An estimation of the relative error produced $\delta\bar{h}$ can now be obtained,

$$\delta\bar{h} = \frac{\bar{h}_1^*-\bar{h}_1}{\bar{h}_0} = \frac{W_1}{W_0}\left|\,1-\left(\frac{W_1}{W_0}\right)^{\alpha-1}\left(\frac{1+K}{1+K(W_1/W_0)^{\alpha}}\right)\right|. \tag{17}$$

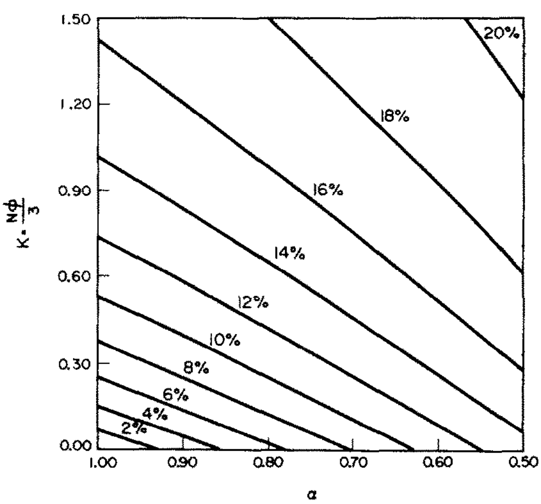

FIG. 1. Dependence of percentage error in BHTC upon α and K. Gas flow rate change = 25%.

In Fig. 1 the percentage error, produced by a 25% increase in gas flow rate (large by most operating conditions) is shown for values of α and K in the ranges $0.5 \leqslant \alpha \leqslant 1$, $0 \leqslant K \leqslant 1.5$. For regenerators of Cowper stove dimensions K will have a maximum value of 0.25 ($N \leqslant 0.75$, $\phi < 1$) and this corresponds to a maximum error of 10% in the calculated BHTC when the Bohm equation is relevant ($\alpha = 0.8$) and a 15% error when the Kistner formulae are employed. For packed bed regenerators Saunders and Ford [4] found, experimentally, a linear relationship between SHTC and gas velocity. The materials investigated were steel, lead and glass, for metal packing the high conductivity will result in a low value of K, with other materials K will usually have an upper limit of 1.5 which corresponds to a maximum error of approximately 16% (see Fig. 1).

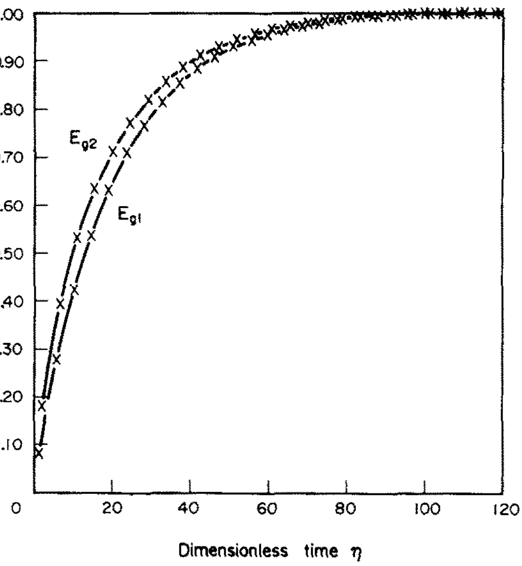

FIG. 2. Responses E_{g1} and E_{g2} to a step change in gas flow rate.

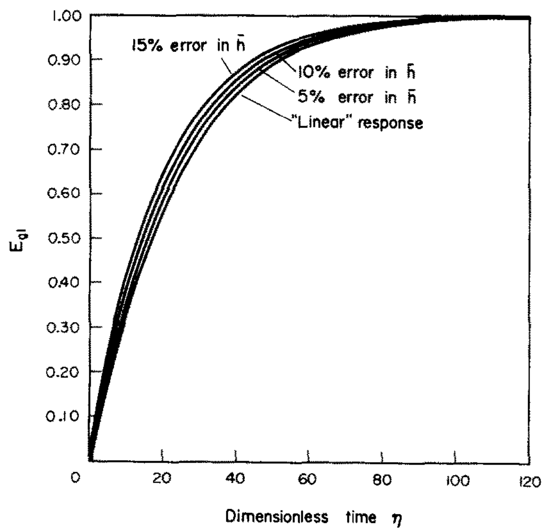

FIG. 3. The effect upon the response E_{g1} of error in the BHTC.

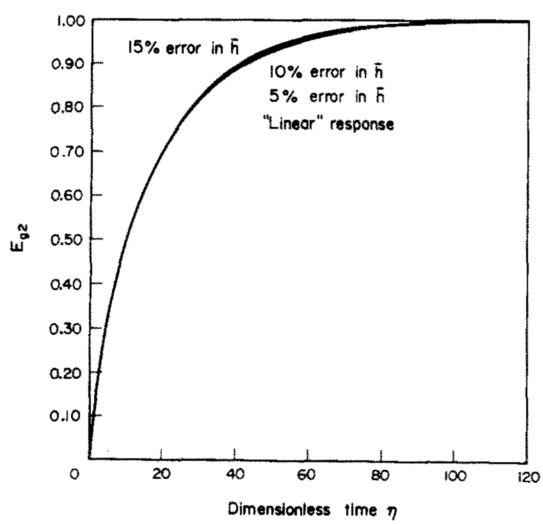

FIG. 4. The effect upon the response E_{g2} of error in the BHTC.

The effect of error in the calculation of BHTC upon transient behaviour

Willmott and Burns [1] illustrated the response of a regenerator to changes in operating conditions by defining the following dimensionless parameters ε_{g1} and ε_{g2} given by

$$\varepsilon_{g1}(n) = \frac{t'^{(n)}_{\mathrm{OUT}} - t'^{(0)}_{\mathrm{OUT}}}{t'^{(\infty)}_{\mathrm{OUT}} - t'^{(0)}_{\mathrm{OUT}}} \tag{18}$$

$$\varepsilon_{g2}(n) = \frac{t''^{(n)}_{\mathrm{OUT}} - t''^{(0)}_{\mathrm{OUT}}}{t''^{(\infty)}_{\mathrm{OUT}} - t''^{(0)}_{\mathrm{OUT}}} \tag{19}$$

where $0 \leqslant n \leqslant \infty$.

$t'^{(n)}_{\mathrm{OUT}}$ and $t''^{(n)}_{\mathrm{OUT}}$ denote the mean exit gas temperature for the nth cycle following the step change. At equilibrium, immediately prior to a step change ($n = 0$), $\varepsilon_{g1}(0) = \varepsilon_{g2}(0) = 0$; once cyclic equilibrium has been restored ($n = \infty$), $\varepsilon_{g1}(\infty) = \varepsilon_{g2}(\infty) = 1$.

In Fig. 2 typical responses ε_{g1} and ε_{g2} (starred points) are illustrated, they were produced by a 25% step change in gas flow rate to a regenerator of Cowper stove dimensions. In order to facilitate clearer graphical presentation the continuous smooth curves E_{g1} and E_{g2} are also produced in Fig. 2. These curves are defined as a series of cubic polynomials through successive ε_{g1} and ε_{g2} data points.

The effect upon final exit temperatures $t'^{(\infty)}_{\mathrm{OUT}}$ and $t''^{(\infty)}_{\mathrm{OUT}}$ of any inaccuracy in the calculated BHTC was found to be negligible being less than 1% for even large (20%) errors in $\bar{h}$. However the transient responses ε_{g1} and ε_{g2} do exhibit some dependence upon the induced error in BHTC. In Figs. 3 and 4 the curves E_{g1} and E_{g2} are produced for 0, 5, 10 and 15% errors in $\bar{h}$ following a 25% step increase in hot gas flow rate. It is evident from these figures that the more important cold side response exhibits far less dependence upon this error in $\bar{h}$ than does the hot side response.

In order to give a measure of the distinction between the "correct" curves, E^*_{g1} and E^*_{g2} and the ones obtained by a P% error in BHTC, E^P_{g1} and E^P_{g2} over a wide range of regenerator parameters the following percentage measure is defined.

$$L'_i = 100^* \int_0^\Theta (E^*_{gi} - E^P_{gi})\,\mathrm{d}\theta \Big/ \int_0^\Theta E^*_{gi}\,\mathrm{d}\theta, \tag{20}$$

for $i = 1, 2$,

where Θ is the total dimensionless time required by the regenerator to re-establish cyclic equilibrium as defined by the convergence criteria described in the appendix of Willmott and Burns [1]. For the purpose of computation the integrals in equation (20) are approximated by Gregory's formula [10] that is:

$$\int_0^\Theta E_{gi}\,\mathrm{d}\theta \approx \frac{\Theta}{S}\left\{\frac{1}{2}\varepsilon_{gi}(0) + \sum_{r=1}^{s-1}\varepsilon_{gi}(r) + \frac{1}{2}\varepsilon_{gi}(s) - \frac{1}{12}[\nabla\varepsilon_{gi}(s) - \Delta\varepsilon_{gi}(0)] - \frac{1}{24}[\nabla^2\varepsilon_{gi}(s) + \Delta^2\varepsilon_{gi}(0)] - \frac{19}{720}[\nabla^3\varepsilon_{gi}(s) - \Delta^3\varepsilon_{gi}(0)]\right\},$$

where Δ and ∇ are the forward and backward differences respectively and s is the number of cycles of operation in the transient phase.

Represented in Tables 1 and 2 are the magnitudes of L'_1 and L''_1 obtained by computer simulation for values of reduced length $\Lambda = 1, 5, 10$ and 15 and of reduced period Π given by $\Lambda/\Pi = 1, 5$ and 10. The magnitude of the flow rate change was again 25% and the percentage errors in BHTC investigated were 2, 5, 10 and 15%.

CONCLUSION

It is clear from Table 2 that only for exceedingly small values of reduced length ($\Lambda < 1$) would there be any significant difference between the real cold side response and the one obtained by the simplified model. However the hot side approximation is not acceptable in the circumstances where a large perturbation in BHTC (> 10%) combines with a regenerator which has small values of reduced length ($\Lambda < 5$) and reduced period ($\Pi < 1$). With this size of regenerator, however, the relative inefficiency of heat transfer usually prohibits its use in industrial applications. Cowper stoves for example have reduced lengths in the range $10 \leqslant \Lambda \leqslant 15$ which when combined with the use of the Bohm or even the Kistner formulae for surface heat-transfer coefficient enables acceptable approximations for the regenerator's transient behaviour to be produced.

Table 1. Percentage differences L'_1

		% error in $\bar{h}$			
Λ	Λ/Π	2	5	10	15
	10	3.338	7.914	16.634	27.744
1	5	2.149	5.688	14.098	22.934
	1	1.198	3.124	6.727	10.907
	10	1.669	2.681	3.987	6.446
5	5	1.062	2.013	3.615	5.546
	1	0.238	0.593	1.179	1.757
	10	0.355	1.246	2.513	3.803
10	5	0.342	0.847	2.494	3.319
	1	0.118	0.292	0.571	0.840
	10	0.251	0.890	2.076	2.980
15	5	0.238	0.587	1.748	2.958
	1	0.080	0.196	0.382	0.558

Table 2. Percentage differences L''_1

		% error in $\bar{h}$			
Λ	Λ/Π	2	5	10	15
	10	0.732	0.977	1.112	1.283
1	5	−0.140	−0.341	0.133	−0.164
	1	−0.091	−0.222	0.423	−0.609
	10	−0.127	−0.308	−0.146	0.053
5	5	0.378	0.217	−0.021	0.551
	1	−0.046	−0.112	−0.211	−0.300
	10	−0.100	0.038	0.107	0.204
10	5	−0.095	−0.227	0.139	−0.044
	1	−0.033	−0.080	−0.153	−0.219
	10	−0.084	−0.007	0.213	0.250
15	5	−0.080	−0.191	0.041	0.305
	1	−0.028	−0.068	−0.129	−0.186

REFERENCES

1. A. J. Willmott and A. Burns, The transient response of periodic-flow regenerators, *Int. J. Heat Mass Transfer* **20**, 753–761 (1977).
2. A. Burns and A. J. Willmott, The transient performance of periodic-flow regenerators, *Int. J. Heat Mass Transfer* **21**, 623–627 (1978).
3. J. Schofield, P. Butterfield and P. A. Young, Hot blast stoves, *J. Iron Steel Inst.* **199**, 229–240 (1961).
4. O. A. Saunders and H. Ford, Heat transfer in the flow of gas through a bed of solid particles, *J. Iron Steel Inst.* **141**, 291–316 (1940).
5. W. H. McAdams, *Heat Transmission*. McGraw-Hill, New York (1954).
6. W. M. Kays and A. L. London, *Compact Heat Exchangers*. McGraw-Hill, New York (1964).
7. H. Hausen, Über die Theorie des Wärmeaustausches in

regeneratoren, *Z. Angew. Math. Mech.* **3**, 173–200 (1929).
8. A. J. Willmott and R. J. Thomas, Analysis of the long contra-flow regenerative heat exchanger, *J. Inst. Math. Applic.* **14**, 267–280 (1974).
9. H. Hausen, Vervollstandigte Berechnung des Wärmeaustauches in Regeneratoren, *Z. Ver. Dt. Ing, Beihelft Verftk* **2**, 31–43 (1942).
10. D. R. Hartree, *Numerical Analysis*, p. 114. Oxford University Press, Oxford (1958).

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MEASUREMENT OF BOILING CURVES DURING REWETTING OF A HOT CIRCULAR DUCT

W. J. CHEN,* Y. LEE* and D. C. GROENEVELD†

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NOMENCLATURE

c_p, specific heat at constant pressure [J/kg K];
D, diameter [m];
G, mass flow rate [g/m² s];
h, heat-transfer coefficient [W/m² K];
i, enthalpy [J/kg];
k, thermal conductivity [W/m K];
Q, heat [W];
q'', heat flux [W/m²];
q''', heat generation rate [W/m³];
T, temperature [°C];
t, time;
U, rewetting velocity [m/s];
w, flow rate [kg];
z, length [m];
ρ, density [kg/m³].

Subscripts

a, heat conducted axially;
ac, heat convected to the coolant;
el, electrical heat generation;
i, inside;
in, inlet;
ls, heat loss to surroundings;
o, outside;
p, peripheral;
q, quenching;
rd, net radiation from the wall to the coolant;
sat, saturation;
w, wall;
wi, wall inside;
wo, wall outside;
z, elevation.

INTRODUCTION

THE PRESENT study was designed to investigate transient heat-transfer modes which can be encountered during reflooding of a reactor core following a loss-of-coolant accident. Previous reflooding studies have concentrated on (i) developing analytical models for reflooding [1–3] which are based on assumptions regarding heat-transfer coefficients on the wet and dry side, and (ii) experimental studies which were mainly concerned with measurement of rewetting velocities and wall temperature–time gradients [4]. The contribution of this study is that it presents experimentally derived boiling curves obtained during reflooding tests over a wide range of experimental parameters.

*Department of Mechanical Engineering, University of Ottawa, Ottawa, Canada K1N 6N5.
†Atomic Energy of Canada Limited, Chalk River, Ontario, Canada K0J 1J0.

EXPERIMENT

The experimental loop consisted of the following components in series: demineralizer, preheater, boiler, pump, flowmeter, test section with parallel bypass, and condenser. Three test sections were used in this investigation; details are presented in Table 1.

Each test section was instrumented with about forty chromel–alumel thermocouples, spot-welded onto the outside wall surface at different axial and peripheral positions. The test sections were heated by a 30 kVA, 1500 amp AC power supply and were well insulated. The terminal clamps, made of copper, were attached to both ends of the test section. Pressure transducers were used for monitoring test section inlet and outlet pressure. Further details are given in [5].

All tests were conducted using water at atmospheric pressure, with the following ranges of test parameters: flow rate 10–40 g cm^{-2} s^{-1}, initial wall temperature 270–800°C, inlet subcooling 10–80°C, test section power 0–20 kW. At the start of each test the power to the test section was turned on and the wall temperature was brought up to the desired value. Experiments were performed by diverting the water flow from the bypass circuit, to the test section while the power was either maintained or switched off, depending on the selection of the test parameters. The signals from the thermocouples, and from the transducers (for flow rate, pressures, and power to the test section) were recorded and printed out simultaneously.

DATA REDUCTION

The rewetting velocities and the average initial wall temperatures were derived from the temperature–time traces using the method commonly used by other investigators [6, 7].

The surface heat flux to the coolant may be extracted from the data by using the energy balance for the length ΔZ of the test section shown in Fig. 1

$$\frac{dQ}{dt} = Q_{el} - Q_c - Q_a - Q_{rd} - Q_{ls}. \quad (1)$$

It is assumed here that the axial conduction heat flux may be approximated by a one-dimensional conduction model. Justification will be provided in the discussion section.